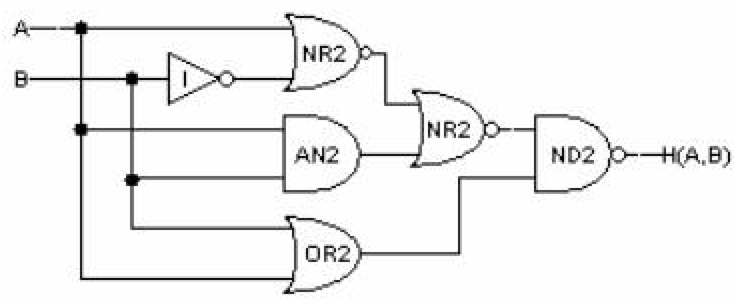
Show All Answers Hide All Answers

Gates and Boolean logic

★ indicates problems that have been selected for discussion in section, time permitting.

Problem 1. Consider the following circuit that implements the 2-input function H(A,B):



A. ★ Fill in the following truth table for H:

A	B	H
0	0	
0	1	
1	0	
1	1	

[redacted]

B. ★ Give a sum-of-products expression that corresponds to the truth table above.

[redacted]

[redacted]

C. ★ Using the following table of timing specifications for each component, what are t_{CD}, t_{PD} and t_R for the circuit shown above?

gate	t_{CD}	t_{PD}	t_R	t_F
I	3ps	15ps	8ps	5ps
ND2	5ps	30ps	11ps	7ps
AN2	12ps	50ps	13ps	9ps
NR2	5ps	30ps	7ps	11ps
OR2	12ps	50ps	9ps	13ps

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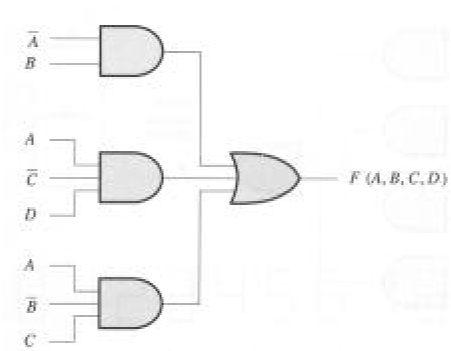
Problem 2. **Gates and Boolean equations**

A. Show the Boolean equation for the function F described by the following circuit:



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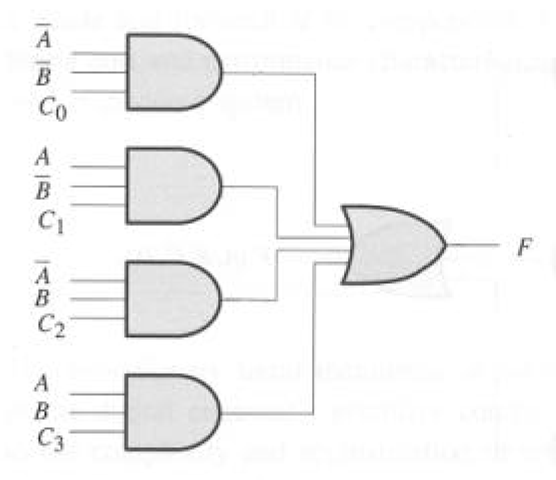
B. ★ Consider the circuit shown below. Each of the control inputs, C0 through C3, must be tied to a constant, either 0 or 1.



What are the values of C0 through C3 that would cause F to be the *exclusive OR* of A and B?

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C. ★ Can any arbitrary Boolean function of A and B be realized through appropriate wiring of the control signals C0 through C3?

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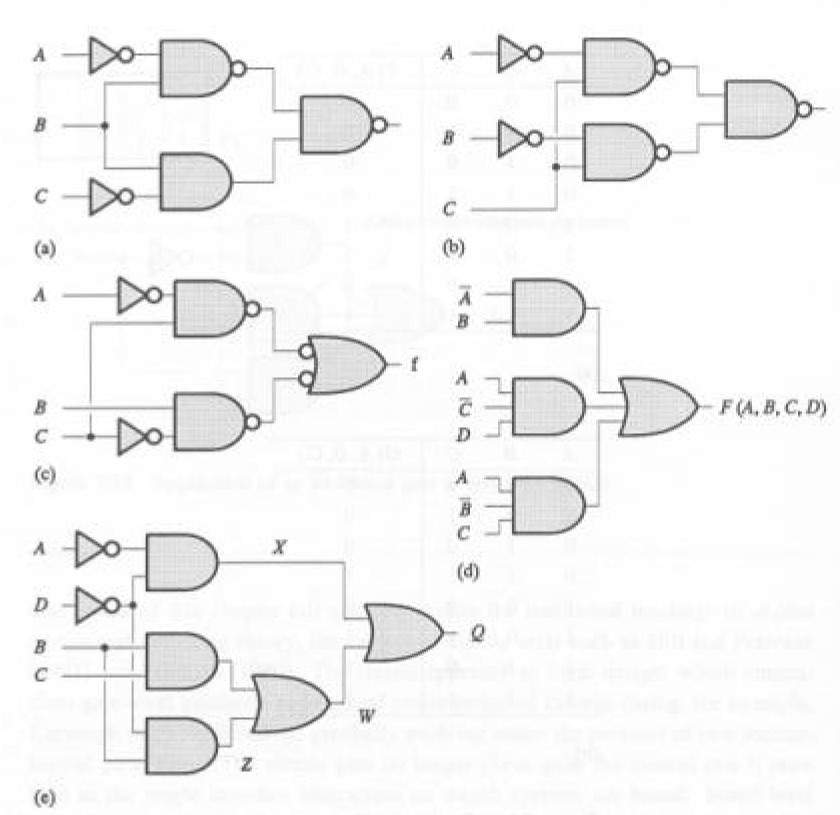
D. Give a sum -of-products expression for each of the following circuits:



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E. Give a canonical sum-of-products expression for the Boolean function described by each truth table below

A	B	C	$F(A,B,C)$
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

A	B	C	$G(A,B,C)$
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

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[redacted]

[redacted]

F. We've seen that there are a total of sixteen 2-input Boolean functions. How many 5-input Boolean functions are there?

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Problem 3. A priority encoder has inputs that are assigned some predetermined order. The output is the binary encoding of the first "1" valued input from the ordered list, and it is zero otherwise.

A. ★ Give the truth table for a 3-input priority encoder.

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B. ★ Give a sum of products realization of this priority encoder.

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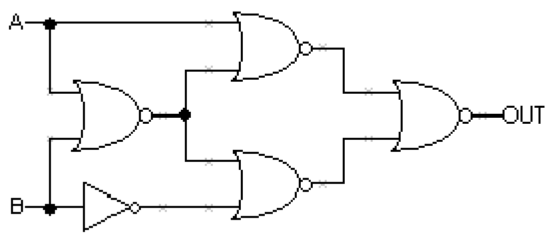
Problem 4. Suppose we are building circuits using only the following three components:

- inverter: tcd = 0.5ns, tpd = 1.0ns, tr = tf = 0.7ns
- 2-input NAND: tcd = 0.5ns, tpd = 2.0ns, tr = tf = 1.2ns
- 2-input NOR: tcd = 0.5ns, tpd = 2.0ns, tr = tf = 1.2ns

Consider the following circuit constructed from an inverter and four 2-input NOR gates:



A. ★ What is t_{PD} for this circuit?

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B. ★ What is t_{CD} for this circuit?

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C. ★ What is the output rise time for this circuit?

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D. ★ What is t_{PD} of the *fastest* equivalent circuit (i.e., one that implements the same function) built using only the three components listed above?

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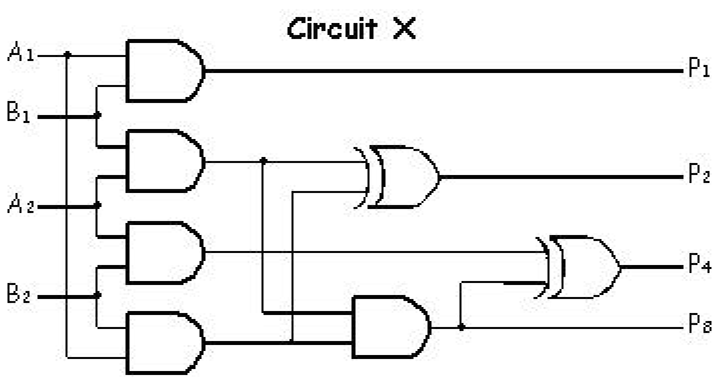
Problem 6. **The Mysterious Circuit X**

A. Determine the function of the Circuit X, below, by writing out and examining its truth table. Give a minimal sum -of- products Boolean expression for each output.



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A_2	A_1	B_2	B_1	P_8	P_4	P_2	P_1
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0
0	0	1	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	0	0	0	0	0
0	1	0	1	0	0	0	1
0	1	1	0	0	0	1	0
0	1	1	1	0	0	1	1
1	0	0	0	0	0	0	0
1	0	0	1	0	0	1	0
1	0	1	0	0	1	0	0
1	0	1	1	0	1	1	0
1	1	0	0	0	0	0	0
1	1	0	1	0	0	1	1
1	1	1	0	0	1	1	0
1	1	1	1	1	0	0	1

[redacted]

B. For Circuit X assume that AND gates have a propagation of 2 nS and a contamination delay of 1nS, while XOR gates have a propagation delay of 3 nS and contamination delay of 2 nS.

Compute the aggregate contamination and propagation delays for Circuit X. What is the maximum frequency that the inputs of Circuit X be changed while insuring that all outputs are stable for 5 nS?

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[redacted]

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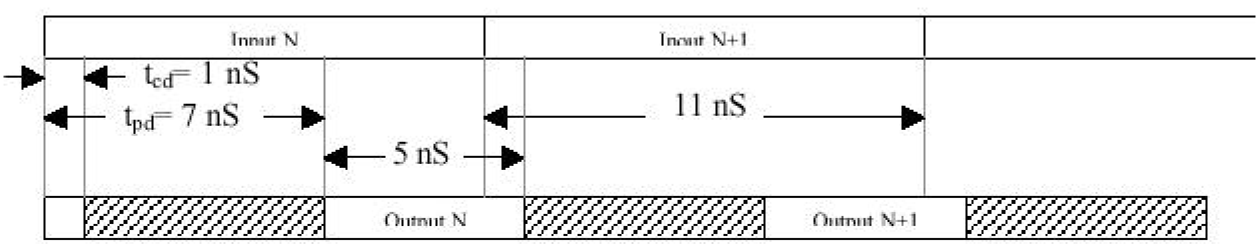

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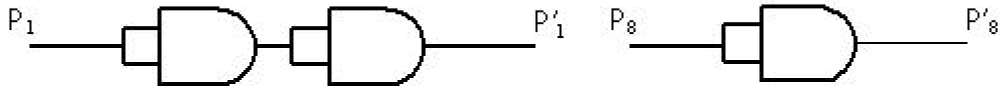
C. Suppose the gates below are added to Circuit X. How are the answers to part b) affected?



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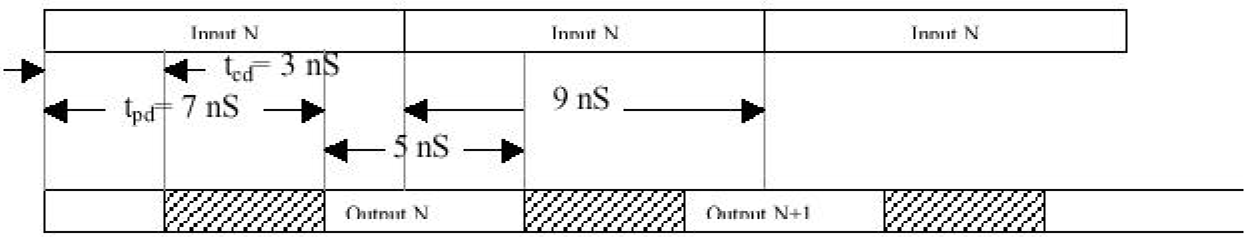

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Problem Set 1

Issued: February 8, 2006 **Due**: February 21, 2006

Boolean Algebra Practice Problems (do not turn in):

Simplify each expression by algebraic manipulation. Try to recognize when it is appropriate to transform to the dual, simplify, and re-transform (e.g. no. 6). Try doing the problems before looking at the solutions which are at the end of this problem set.

1) $a + 0 =$

2) $\bar{a} \cdot 0 =$

3) $a + \bar{a} =$

4) $a + a =$

5) $a + ab =$

6) $a + \bar{a}b =$

7) $a(\bar{a} + b) =$

8) $ab + \bar{a}b =$

9) $(\bar{a} + \bar{b})(\bar{a} + b) =$

10) $a(a + b + c + ...) =$

For (11),(12), (13), $f(a,b,c) = a + b + c$

11) $f(a,b,ab) =$

12) $f(a,b,\bar{a} \cdot \bar{b}) =$

13) $f[a,b,\overline{(ab)}] =$

14) $y + y\bar{y} =$

15) $xy + x\bar{y} =$

16) $\bar{x} + y\bar{x} =$

17) $(w + \bar{x} + y + \bar{z})y =$

18) $(x + \bar{y})(x + y) =$

19) $w + [w + (wx)] =$

20) $x[x + (xy)] =$

21) $\overline{(\bar{x} + \bar{x})} =$

22) $\overline{(x + \bar{x})} =$

23) $w + (w\bar{x}yz) =$

24) $\bar{w} \cdot \overline{(wxyz)} =$

25) $xz + \bar{x}y + zy =$

26) $(x + z)(\bar{x} + y)(z + y) =$

27) $\bar{x} + \bar{y} + xy\bar{z} =$

Problem 1: Karnaugh Maps and Minimal Expressions

For each of the following Boolean expressions, give:

i) The truth table,
ii) The Karnaugh map,
iii) The MSP expression, (Show groupings)
iv) The MPS expression. (Show groupings)

1) $(\bar{a} + b \cdot \bar{d}) \cdot (c \cdot b \cdot a + \bar{c} \cdot d)$

2) $\overline{(w \cdot \bar{x} + y \cdot \bar{z} + \bar{y} \cdot \bar{w} \cdot x)}$

Problem 2: Karnaugh Maps with "Don't Cares"

Karnaugh Maps are useful for finding minimal implementations of Boolean expressions with only a few variables. However, they can be a little tricky when "don't cares" (X) are involved. Using the following K-Maps:

cd \ ab	00	01	11	10
00	X	0	0	1
01	1	0	0	X
11	0	X	0	1
10	0	0	0	1

(1)

cd \ ab	00	01	11	10
00	1	0	0	1
01	0	1	X	1
11	X	1	0	0
10	1	1	0	X

(2)

i) Find the minimal sum of products expression. Show your groupings.
ii) Find the minimal product of sums expression. Show your groupings.
iii) Are your solutions unique? If not, list and show the other minimal expressions.
iv) Does the MPS = MSP?

Problem 3: DeMorgan's Theorem

Use DeMorgan's Theorems to simplify the following expressions:

1) $\overline{\overline{(a+d)}\cdot\overline{(\bar{b}+c)}}$

2) $\overline{\overline{(a\cdot b\cdot\bar{c})}+\overline{(\bar{c}\cdot d)}}$

3) $\overline{\bar{a}+d}\cdot\overline{b+\bar{c}}\cdot\overline{\bar{c}+d}$

Problem 4: Transistor/Gate Level Synthesis

1) Construct a transistor level circuit of the following function using NMOS and PMOS devices: $F=\overline{A\cdot(B+C)}$
2) Construct a gate level circuit of the same function only using NAND gates.

Solutions to the Boolean Algebra Practice Problems

1) $a + 0 = a$
2) $\bar{a} \cdot 0 = 0$
3) $a + \bar{a} = 1$
4) $a + a = a$
5) $a + ab = a(1 + b) = a$
6) $a + \bar{a}b = (a + \bar{a})(a + b) = a + b$
7) $a(\bar{a} + b) = a\bar{a} + ab = ab$
8) $ab + \bar{a}b = b(a + \bar{a}) = b$
9) $(\bar{a} + \bar{b})(\bar{a} + b) = \bar{a}\bar{a} + \bar{a}b + \bar{b}\bar{a} + \bar{b}b = \bar{a} + \bar{a}b + \bar{a}\bar{b} = \bar{a}(1 + b + \bar{b}) = \bar{a}$
10) $a(a + b + c + ...) = aa + ab + ac + ... = a + ab + ac + ... = a$
11) $f(a, b, ab) = a + b + ab = a + b$
12) $f(a, b, \bar{a} \cdot \bar{b}) = a + b + \bar{a}\bar{b} = a + b + \bar{a} = 1$
13) $f[a, b, \overline{(ab)}] = a + b + \overline{(ab)} = a + b + \bar{a} + \bar{b} = 1$
14) $y + y\bar{y} = y$
15) $xy + x\bar{y} = x(y + \bar{y}) = x$
16) $\bar{x} + y\bar{x} = \bar{x}(1 + y) = \bar{x}$
17) $(w + \bar{x} + y + \bar{z})y = y$
18) $(x + \bar{y})(x + y) = x$
19) $w + [w + (wx)] = w$
20) $x[x + (xy)] = x$
21) $\overline{(\bar{x} + \bar{x})} = x$
22) $\overline{(x + \bar{x})} = 0$
23) $w + (w\bar{x}yz) = w(1 + \bar{x}yz) = w$
24) $\bar{w} \cdot \overline{(wxyz)} = \bar{w}(\bar{w} + \bar{x} + \bar{y} + \bar{z}) = \bar{w}$
25) $xz + \bar{x}y + zy = xz + \bar{x}y$
26) $(x + z)(\bar{x} + y)(z + y) = (x + z)(\bar{x} + y)$
27) $\bar{x} + \bar{y} + xy\bar{z} = \bar{x} + \bar{y} + \bar{z}$